Octal Numbers

The Na'vi use an octal, or base-8, number system. Before we get into the vocabulary, let's take a look at the differences between a base-8 system and our familiar base-10 system.

Let's take an example base-10 number: 8946. The way to look at a number in a number system is to take each place as the base raised to the power of the place minus 1. Here's what I mean by that: the number above says that there are 8 of 10^3 (8 is in the 4th place), 9 of 10^2, 4 of 10^1, and 6 of 10^0. Recall that 10^n = 10*10*10*10… n times, so 10^3=10*10*10=1000, and so on (10^0=1). So the number is:

$$8*10^3+9*10^2+4*10^1+6*10^0 = 8000+900+40+6 = 8946$$

Now to do this in base-8, we just treat each place in powers of 8 instead. Take the octal number ^{o}4263. (Notice we denote octal numbers with an "o" in superscript). So, in base-10 representation, this is saying we have 4 of 8^3, 2 of 8^2, 6 of 8^1, and 3 of 8^0. We can convert this to the more-familiar base-10 by multiplying these out and adding them:

$$^o4263 = 4*8^3+2*8^2+6*8+3*1 = 4*512+2*64+6*8+3 = 2048+128+48+3 = 2227$$

What if we want to convert a base-10 number into octal though? The simplest way is to perform the repeated division method. Here's how this works:

Take our number (8946) and divide by 8 (1118 R 2). Take the remainder (2); this is the first position of the octal representation. Then divide the leftover number (1118) by 8 again (139 R 6). The remainder of this is the second position of the octal representation, so we now have ^{o}____62. Repeat this until you have no more divisions. Doing so we get that 8946 in octal is ^{o}21362.

Alternatively, you can check the highest power of eight that will fit within your target number, and from there find the highest multiple of that number that will still fit within the target number. The multiple is your first position of your octal conversion. Subtract the number multiple you got, take the next highest power of eight and repeat. Here's this method in action:

8946
= 2*4096 + 754
= 2*4096 + 1*512 + 242
= 2*4096 + 1*512 + 3*64 + 50
= 2*4096 + 1*512 + 3*64 + 6*8 + 2*1
= $2*8^4 + 1*8^3 + 3*8^2 + 6*8^1 + 2*8^0$
= ^{o}21362

To do these conversions using a computer, the website WolframAlpha.com is great for this. simply type in the number followed by "decimal to octal" or "octal to decimal" depending on which conversion you need. Dict-Na'vi will also do decimal to octal conversions, but it won't do the reverse. Learn Na'vi also has a number converter here.

Now that we've gone through all of that, the vocabulary is as follows:

0-7							
kew	'aw	mune	pxey	tsìng	mrr	pukap	kinä

$8 = 8^1 = °10$	$64 = 8^2 = °100$	$512 = 8^3 = °1000$	$4096 = 8^4 = °10000$
vol	zam	vozam	zazam

To make numbers like 16 = 2*8, we use prefix versions of the 2-7 numbers:

2	3	4	5	6	7
me-	pxe-	tsì-	mrr-	pu-	ki-

When 1-7 come at the end of the number, the word lenits and may shorten. Here are all of them (note that when they come after **vol** in any form, the **l** disappears, except for -aw):

1	2	3	4	5	6	7
-aw	-mun	-pey	-sìng	-mrr	-fu	-hin

This lenited form applies to all smaller numbers, but for some larger numbers the lenition is only applied to the last number word in the sequence. Also, for smaller numbers the **m** in **zam** disappears for the sake of word flow.

An example:

zasìvosìng = **zam tsìvo tsìng** = °144 = 64+4*8+4 = 100

The largest possible number we have is:

kizazamkivozamkizamkivohin = **kizazam kivozam kizam kivo kin** = °77777
= 7*4096+7*512+7*64+7*8+7 = 32767

If you need to refer to the decimal numbers 8 and 9, you can say **'eyt** and **nayn**, but these have no numerical meaning in Na'vi; they just refer to the 8 and 9 symbols.

Practice

Convert the following numbers either from decimal to octal or from octal to decimal:

9	
70	
	º624
	º1267
3054	
	º14624
9954	
15282	
	º64245
	º76233

Express the octal numbers above in the Na'vi number vocabulary:

Octal Arithmetic

Octal arithmetic works surprisingly like normal decimal arithmetic you are familiar with, it's just that you carry over from 8s instead of 10s. For example, 5+5=12 in octal, or 14 + 23 = 37. The trick is to remember that 14 in octal is not the same as 14 in decimal! 14 + 23 = 37 is true in both cases, but it's not that it's the *same quantity* across both systems! The same is true for subtraction: 25 - 12 = 13 and 63 - 27 = 34.

Multiplication and division are slightly trickier, but work off of the same principle. Think of multiplication as repeated addition and division as repeated subtraction. That is: multiplication is "what happens if we add X to X Y number of times?" and division is "how many times can we take Y from X before we reach a number less than Y?" If you think of it in this manner, basic multiplication and division is much easier to grasp!

Some examples:
5 * 2 = 12
12 * 5 = 62
12 * 4 = 50
54 / 4 = 11
12 / 2 = 5
17 / 5 = 3

For more complex problems, unfortunately, we need to learn multiplication tables again. Here's the single multiplication table:

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	1	2	3	4	5	6	7
2	2	4	6	10	12	14	16
3	3	6	11	14	17	22	25
4	4	10	14	20	24	30	34
5	5	12	17	24	31	36	43
6	6	14	22	30	36	44	52
7	7	16	25	34	43	52	61

For problems that require carrying, just remember that you are carrying over an 8 instead of a 10! That is, when counting down you go 11, 10, 7, 6, 5, … instead of 11, 10, 9, 8, 7, …

Practice

Solve the following problems in octal arithmetic:

3 + 4	7 + 6
24 + 12	42 + 47
7 - 5	12 - 5
54 - 32	65 - 46
2 * 3	3 * 5
12 * 2	14 * 5
6 / 2	16 / 7
62 / 5	124 / 25

Expand the multiplication table above to include the first set of tens:

	11	**12**	**13**	**14**	**15**	**16**	**17**
11							
12							
13							
14							
15							
16							
17							

Extra Stuff

Universal Constants in Octal:

Constant	**Decimal**	**Octal**
c (*speed of light*)	299792458 m/s	º2167474112 m/s
h (*Planck's constant*)	$6.62607015 \times 10^{-34}$ J s	later xD